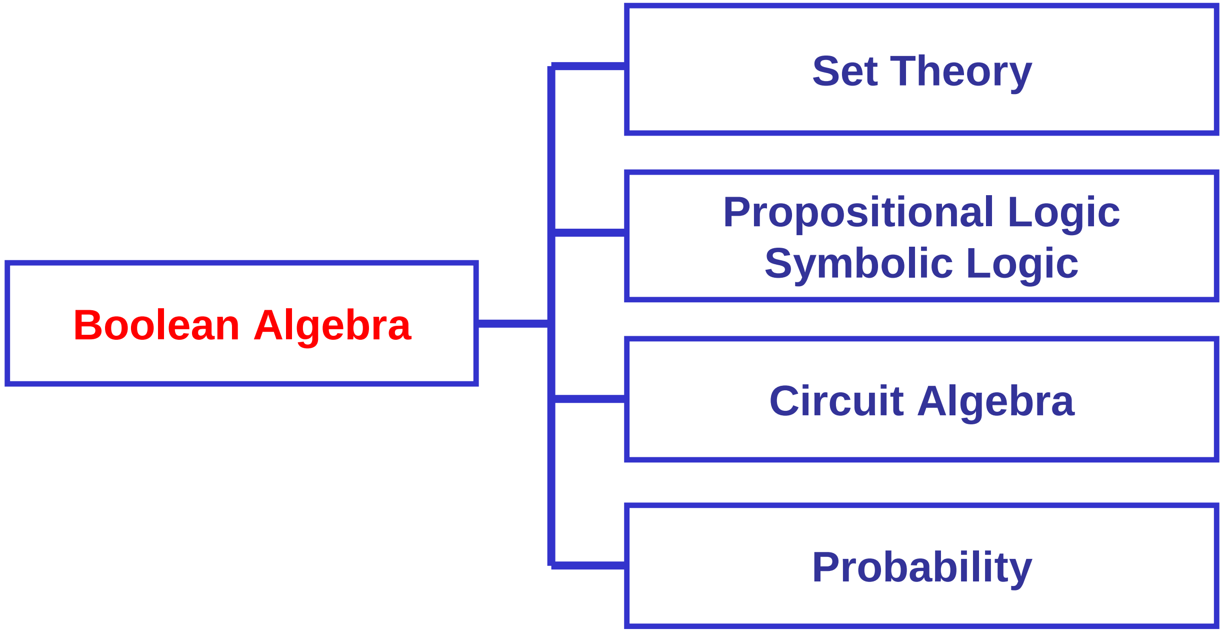
FACHHOCHSCHULE KIEL
University of Applied Sciences

Digital Design

Classification of Boolean Functions

Intro

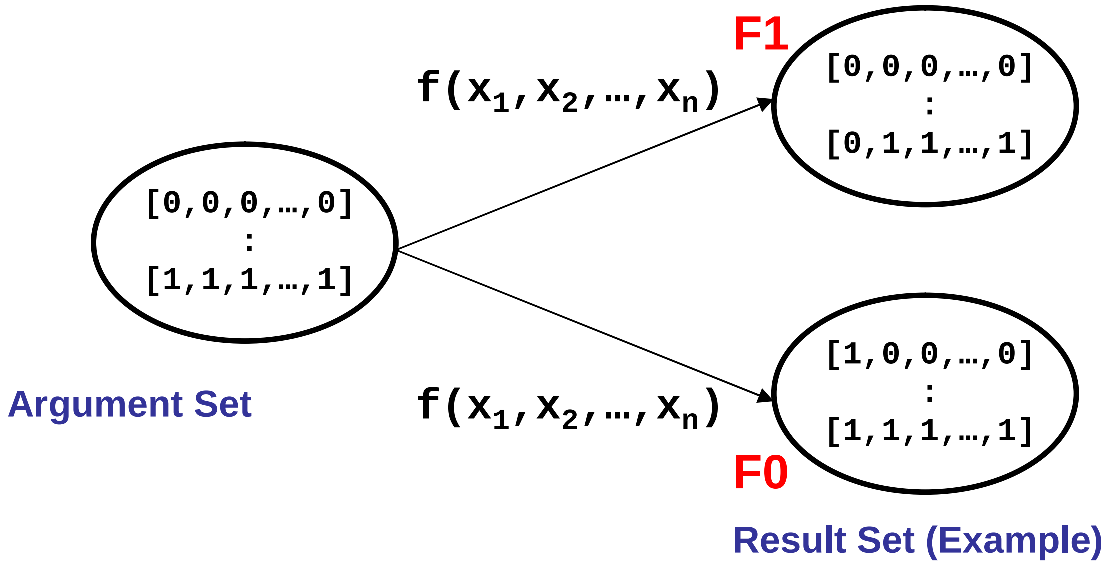
Boolean algebra can now be divided into four areas:



FACHHOCHSCHULE KIEL University of Applied Sciences

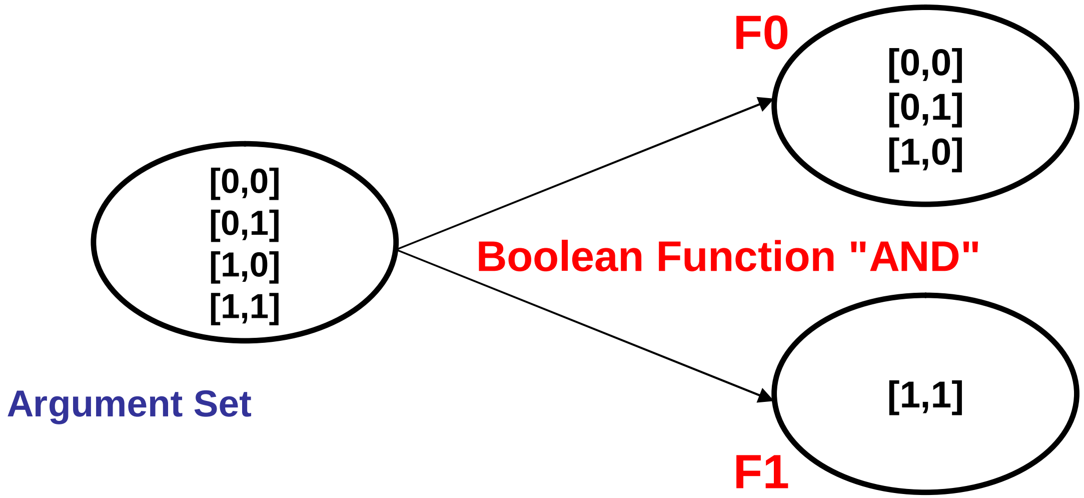
Intro

Definition of a Boolean Function :



Intro

The Boolean Function "AND":

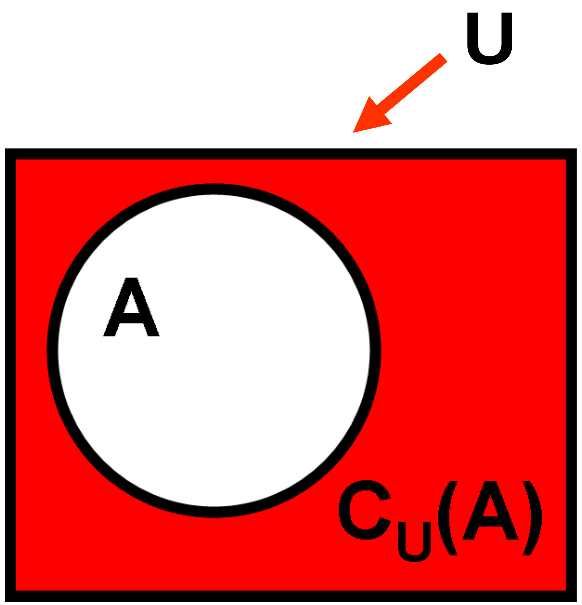


Basic Functions in Digital Design: OR, AND, NOT

Considering the Boolean Functions with n = 1 and n = 2 the functions AND, OR, and NOT are of special importance.

Therefore their properties will be discussed in detail.

The Boolean Function "NOT":



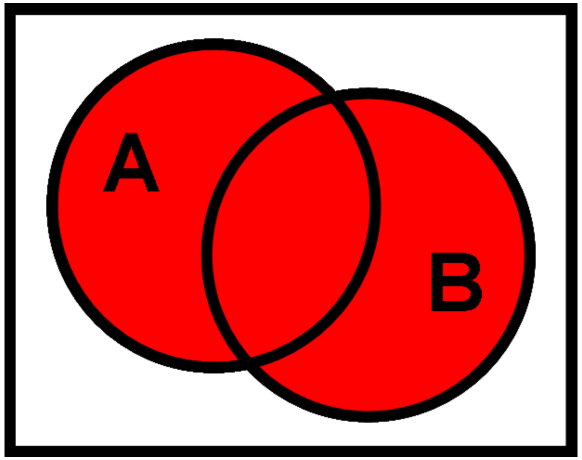
Venn Diagram

Boolean Functions

The Boolean Function "OR":

Also called "Disjunction".

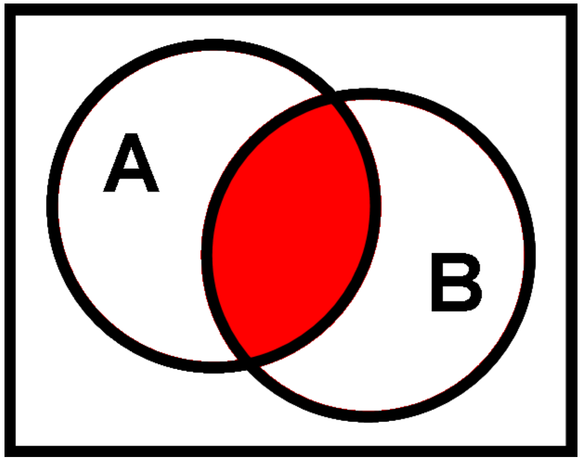
Venn Diagram



The Boolean Function "AND":

Also called "Conjunction".

Venn Diagram



Intro

Considering a Boolean function f with n Arguments x_i, a generic truth table can be defined:

With: $f_i \in \{0,1\}$

x_n	...	x_2	x_1	$f(x_1,x_2,\ldots,x_n)$
0	...	0	0	f_1
0	...	0	1	f_2
0	...	1	0	f_3
0	...	1	1	f_4
:	...	:	:	:
:	...	:	:	:
1	...	1	1	f_{2^n}

Boolean Functions

As can be seen, this table shows in 2^n rows for each of the possible 2^n combinations of the variables x_1, x_2, …, x_n the corresponding function value.

In each row the functional value 0 or 1 is possible, so that

$$m = 2^{(2^n)}$$

non-equivalent functions $f(x_1, x_2, \ldots, x_n)$ exist.

In the most simple case the Boolean function depends only on one variable (so-called unary Boolean function), so that

$$m = 2^{(2^1)} = 2^2 = 4$$ exist.

Boolean Functions

In the case of two variables (binary Boolean functions)

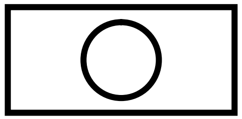
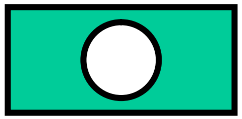
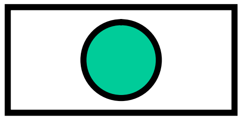
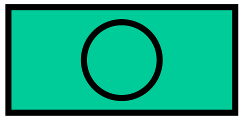
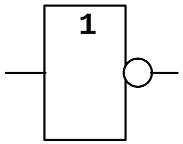
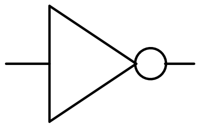
$$m = 2^{(2^2)} = 2^4 = 16$$

As can be seen, the number of possible Boolean functions strongly increases when the number of variables n increases.

Although this number is very big, it will be seen that only some of these functions have non-trivial properties and are therefore important for practical applications.

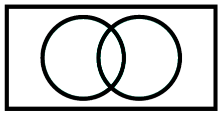
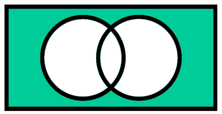
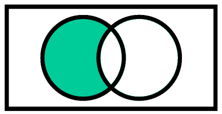
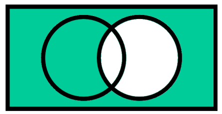
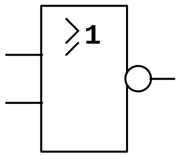
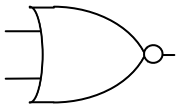
Boolean Functions

Unary Boolean Functions (n = 1)

Name		1-Function	Negation	Identity	0-Function
Truth Table	**a**	f(a)	f(a)	f(a)	f(a)
	0	**0**	**1**	**0**	**1**
	1	**0**	**0**	**1**	**1**
Function		$f = 0$	$f = \bar{a}$	$f = a$	$f = 1$
Venn Diagram					
Graphic Symbol ANSI/IEEE DIN/IEC					
Graphic Symbol					

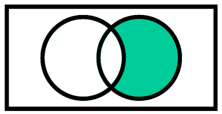
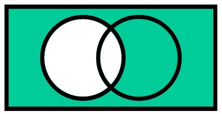
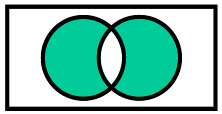
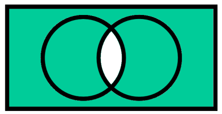
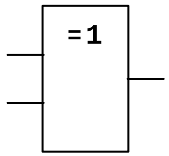
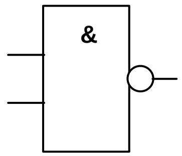
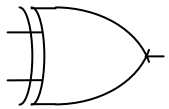
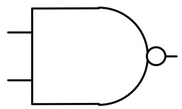
Boolean Functions

Binary Boolean Functions (n = 2) – Part 1

Name			0-Function	NOR		$\overline{b}$
Truth Table	b	a	f(a, b)	f(a, b)	f(a, b)	f(a, b)
	0	0	0	1	0	1
	0	1	0	0	1	1
	1	0	0	0	0	0
	1	1	0	0	0	0
Function			$f = 0$	$f = \overline{a \vee b}$ $f = \overline{a + b}$	$f = a \wedge \overline{b}$ $f = a \cdot \overline{b}$	$f = \overline{b}$
Venn Diagram						
Graphic Symbol ANSI/IEEE DIN/IEC						
Graphic Symbol						

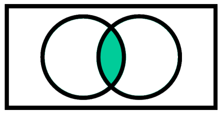
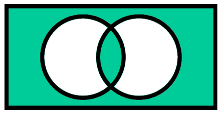
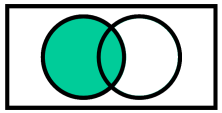
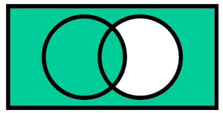
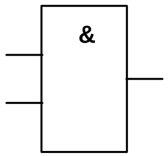
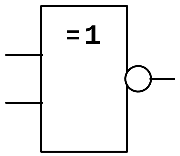
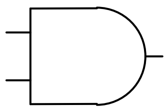
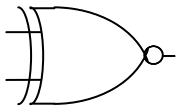
Boolean Functions

Binary Boolean Functions (n = 2) – Part 2

Name				$\bar{a}$	XOR	NAND
Truth Table	b	a	f(a, b)	f(a, b)	f(a, b)	f(a, b)
	0	0	0	1	0	1
	0	1	0	0	1	1
	1	0	1	1	1	1
	1	1	0	0	0	0
Function			$f = \bar{a} \wedge b$ $f = \bar{a} \cdot b$	$f = \bar{a}$	$f = a \oplus b$	$f = \overline{a \wedge b}$ $f = \overline{a \cdot b}$
Venn Diagram						
Graphic Symbol ANSI/IEEE DIN/IEC						
Graphic Symbol						

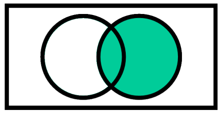
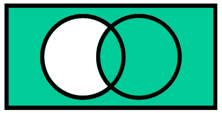
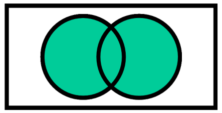
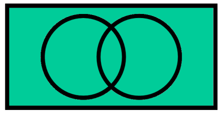
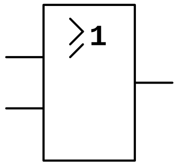
Boolean Functions

Binary Boolean Functions (n = 2) – Part 3

Name			AND	XNOR	a	
Truth Table	b	a	f(a, b)	f(a, b)	f(a, b)	f(a, b)
	0	0	0	1	0	1
	0	1	0	0	1	1
	1	0	0	0	0	0
	1	1	1	1	1	1
Function			$f = a \wedge b$ $f = a \cdot b$	$f = \overline{a \oplus b}$	$f = a$	$f = a \vee \overline{b}$ $f = a + \overline{b}$
Venn Diagram						
Graphic Symbol ANSI/IEEE DIN/IEC						
Graphic Symbol						

Boolean Functions

Binary Boolean Functions (n = 2) – Part 4

Name			b		OR	1-Function
Truth Table	b	a	$f(a,b)$	$f(a,b)$	$f(a,b)$	$f(a,b)$
	0	0	0	1	0	1
	0	1	0	0	1	1
	1	0	1	1	1	1
	1	1	1	1	1	1
Function			$f = b$	$f = \bar{a} \vee b$ $f = \bar{a} + b$	$f = a \vee b$ $f = a + b$	$f = 1$
Venn Diagram						
Graphic Symbol ANSI/IEEE DIN/IEC						
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